Solvay Brussels School Economics & Management

ULB

Financial Risk Management and Governance

Beyond VaR

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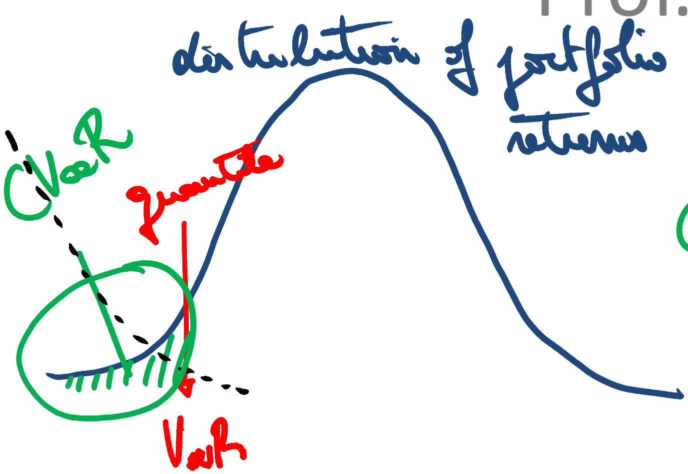


Generally : Spectral Risk measures.

$$CVaR = \frac{\text{WA above VaR}}{1-c}$$

Proba x E(loss)

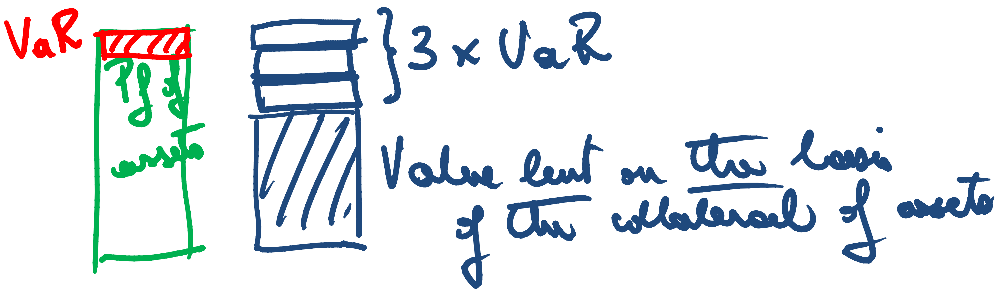
~0 lot/huge

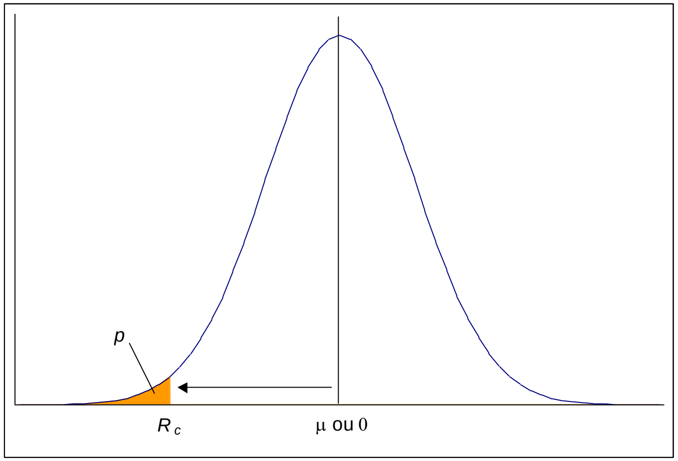
VaR

- Attempt to provide a single number that summarizes the total risk in a portfolio.
- What loss level is such that we are X% confident it will not be exceeded in N business days?"
- Examples: VaR and regulation
  - Regulators base the capital they require banks to keep on VaR
  - The market-risk capital is k times the 10-day 99% VaR where k is at least 3.0
  - Under Basel II capital for credit risk and operational risk is based on a one-year 99.9% VaR

Basel II (Pillar 1) — Standard (Talks) / Advanced approach (market risk --> VaR)

- Is it the only measure we have?





Comparison of models

	Delta-Normal (or var-covar)	Historical Simulation	MonteCarlo Simulation
Valuation	Linear (Local)	Full	Full
Distribution			
▪ Shape	→ Normal	→ Actual	→ General
▪ Extreme events	→ Low probability	→ In recent data	→ Possible
Implementation			
▪ Ease of computation	→ Yes	→ Intermediate	→ No
▪ Communicability	→ Easy	→ Easy	→ Difficult
▪ VaR precision	→ Excellent	→ Poor with short window	→ Good with many iterations
▪ Major pitfalls	→ Non-linearities, fat tails	→ Time variation in risk, unusual events	→ Model risk

Inspired from Jorion, *Financial Risk Manager Handbook*

Alternative Measures of Risk

- Using the entire distribution
  - ✓ Report a range of VaRs for increasing confidence levels
- The conditional VaR (CVaR)
  - ✓ Expected Loss when it is greater than VaR

$$E\left[X \middle| X < VaR_c\right] = \int_{-\infty}^{VaR_c} x\, f(x)\, dx \Big/ \int_{-\infty}^{VaR_c} f(x)\, dx = \int_{-\infty}^{VaR_c} x\, f(x)\, dx \Big/ p$$

outcomes, proba (outcome), because it is conditional

  - ✓ When the value at risk measure falls into a probability mass (i.e., there exists some $\varepsilon > 0$ such that $V_{c+\varepsilon} = V_c$), we use a more general formulation.

$$\text{Find } \beta^* = \max\left\{\beta : V_c = V_\beta\right\}$$

$$CTE_c = \frac{\left(1-\beta^*\right) \mathrm{E}\left[X \middle| X < VaR_c\right] + \left(\beta^* - c\right) VaR_c}{1-c} = \frac{\left(1-\beta^*\right) \mathrm{E}\left[X \middle| X < VaR_c\right] + \left(\beta^* - c\right) VaR_c}{p}$$

= expected shortfall

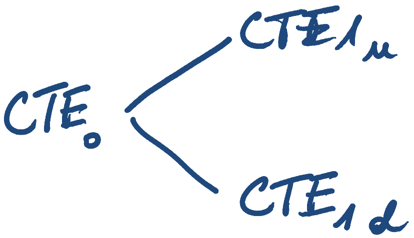
= tail conditional expectation / conditional loss

= expected tail loss / tail risk

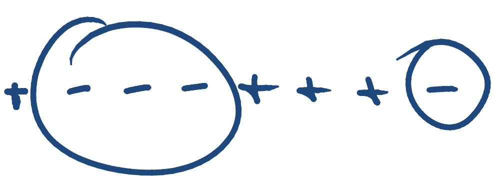
= conditional tail expectation (CTE)

Alternative Measures of Risk (cont'd)

- "Iterated" CTE
  - If CTE has to be revised in the future, prior to the maturity, you may have to provide for more cash if original CTE is upgraded…
  - ICTE proposes to prospectively revaluate the CTE at the future date and aggregate those depending on the probability of each outcome at that future date.



- The standard deviation
  - Covers all observations
  - Is the most efficient measure of dispersion if we stand with normal or Student's t.
  - Var-covar: VaR inherits all properties of standard deviations
  - But symmetrical and cannot distinguish large losses from large gains
  - VaR from SD requires distributional assumption not necessarily valid
- The semi-standard deviation

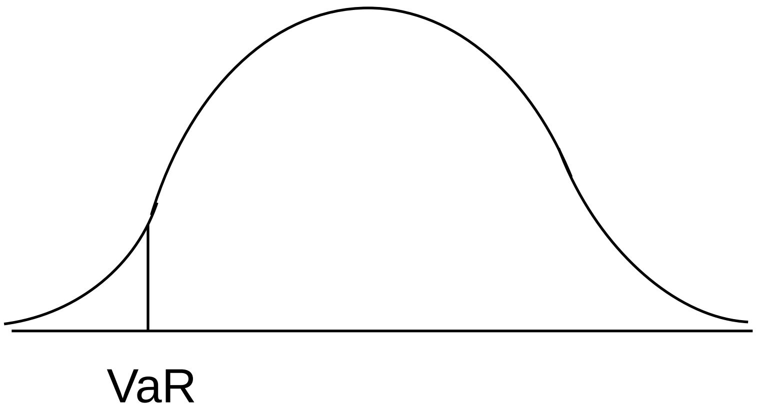


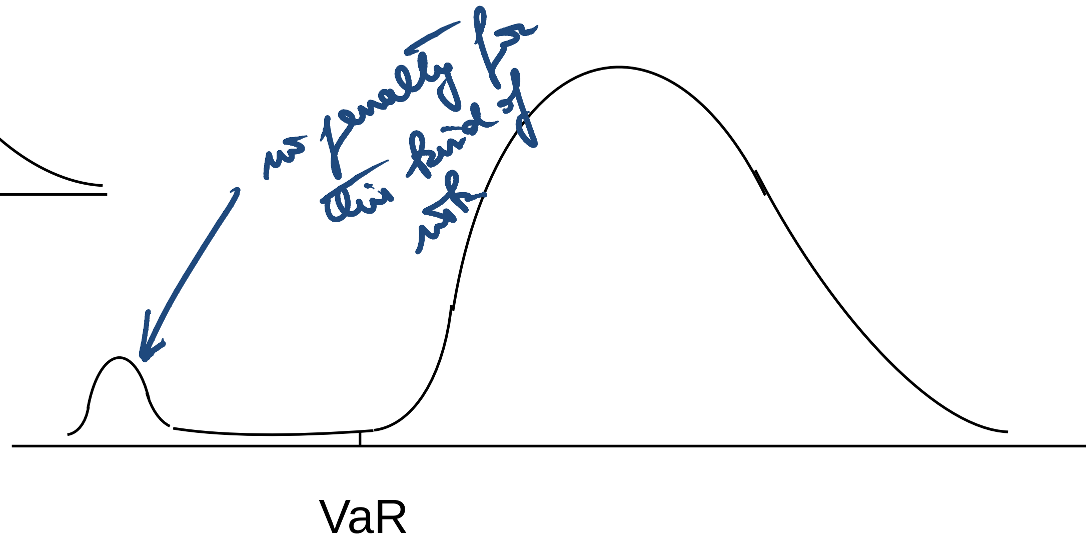
$$s_L = \sqrt{\frac{1}{n_L - 1}\sum_{i=1}^{n}\left[\mathrm{Min}(x_i, 0) - \bar{x}\right]^2}$$

σ of negative cases only
irregularly spread.

Expected shortfall

- Expected shortfall is the expected loss given that the loss is greater than the VaR level (also called C-VaR and Tail Loss)
- Two portfolios with the same VaR can have very different expected shortfalls





ex: + call ATM
= − puts deeply OTM
many

if you apply VaR, not the CVaR.

Source: John Hull, *Risk Management and Financial Institutions.*

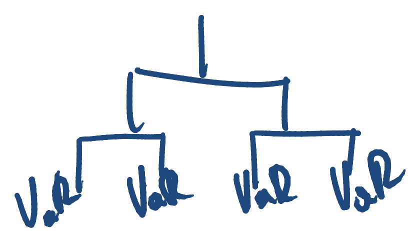
An example

- Case application

Stemming from: Boyle, Hardy & Vorst (2005), “Life after VaR”, *The Journal of Derivatives*.

Coherence of Risk Measures

- Until now:
  - VaR as a downside risk
  - VaR is seen as a quantile
- But:
  - VaR may hide different distribution patterns
  - VaR may be inconsistent for some desirable properties of risk measures
- Desirable properties of a risk measure
  - Monotonicity $if\ X_1 \leq X_2 \rightarrow RM(X_1) \geq RM(X_2)$
  - Translation invariance $RM(X+k) = RM(X) - k$
  - Homogeneity $RM(bX) = b\,RM(X)$
  - Subadditivity (diversification) $RM(X_1 + X_2) \leq RM(X_1) + RM(X_2)$ ⇒ VaR does not guarantee this.
- A risk measure can be characterized by the weights it assigns to quantiles of the loss distribution...



Source: Artzner, Delbaen, Eber & Heath (1999), "Coherent Measures of Risk", *Mathematical Finance*.

Some ideas...

- The expected shortfall
  - Is coherent ✓
  - Gives equal weight to quantiles > q^{th} quantile and 0 to all quantiles < q^{th} quantile
  - Is less simple and harder to back test ✓
- We can also define a *spectral risk measure* by making other assumptions
  - Coherent (satisfies subadditivity) if the weight assigned to q^{th} quantile (w_q) is a nondecreasing function of q.
  - Exponential spectral risk measure

$$w_q = e^{-(1-q)/\gamma}$$

assigning a weighting scheme (≈ penalization scheme) to the distribution of values of the portfolio.

Some parameterizations...

- Sigma, time horizon and VaR

$$N\text{-day VaR} = 1\text{-day VaR} \times \sqrt{N} = 1\text{-day } \sigma N^{-1}(c)$$

  - Ex: Regulatory capital for market risks: $3 \times \sqrt{10} \times 1\text{-day VaR}(99\%)$

- Autocorrelation
  - Changes in portfolio values are not totally independent
  - Assume variance of ΔP_t to be σ^2 for all i, and the correlation between ΔP_t and ΔP_{t-1} (first-order autocorrelation) to be ρ, then

$$\operatorname{var}(\Delta P_t + \Delta P_{t-1}) = (2 + 2\rho)\sigma^2$$

  - Since the correlation between ΔP_t and ΔP_{t-j} is then ρ^j, we have that

$$\operatorname{var}\left(\sum_{j=1}^{N} \Delta P_{t-j}\right) = \sigma^2\left[N + 2(N-1)\rho + 2(N-2)\rho^2 + 2(N-3)\rho^3 + \ldots + 2\rho^{N-1}\right]$$

- Confidence intervals
  - Since it is difficult to estimate VaRs with high confidence intervals directly
    - ✓ We can use a first confidence interval
    - ✓ Then "extrapolate" through the change of confidence interval (but we depend on an assumption on the tails of the distribution)

Backtesting

- Backtesting a VaR calculation methodology involves looking at how often exceptions (loss > VaR) occur:
  - If more than $(1 - c) \rightarrow$ underestimations of VaR
  - If less than $(1 - c) \rightarrow$ overestimations of VaR
- Alternatives:
  - compare VaR with actual change in portfolio value and/or
  - compare VaR with change in portfolio value assuming no change in portfolio composition
- Suppose that the theoretical probability of an exception is p (=1 – c). The probability of m or more exceptions in n days is

$$\sum_{k=m}^{n} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

- Kupiec two-tailed test
  - If the probability of an exception under the VaR model is p and m exceptions are observed in n trials, then

$$-2\ln\left[(1-p)^{n-m} p^m\right] + 2\ln\left[(1-m/n)^{n-m} (m/n)^m\right]$$

  - should have a χ^2 distribution with 1 degree of freedom.

Backtesting – Basle Committee rules

- If number of exceptions in previous 250 days is less than 5 the regulatory multiplier, k, is set at 3
- If number of exceptions is 5, 6, 7, 8 and 9 supervisors may set k equal to 3.4, 3.5, 3.65, 3.75, and 3.85, respectively
- If number of exceptions is 10 or more k is set equal to 4

Bunching & Stress-testing

- Bunching
  - » Bunching occurs when exceptions are not evenly spread throughout the backtesting period
  - » Statistical tests for bunching have been developed
    - ✓ Test for autocorrelation (see slides on "Volatility")
    - ✓ Test statistic suggested by Christofferson

$$-2\ln\left[(1-\pi)^{u_{00}+u_{10}}\,\pi^{u_{01}+u_{11}}\right]+2\ln\left[(1-\pi_{01})^{u_{00}}\,\pi_{01}^{u_{01}}\,(1-\pi_{11})^{u_{10}}\,\pi_{11}^{u_{11}}\right]\sim\chi_1^2$$

u_{ij} is the #obs where we go from a day in state i to a day in state j. State 0 is a day without exception and state 1 is a day with exception.

$$\pi=\frac{u_{01}+u_{11}}{u_{00}+u_{01}+u_{10}+u_{11}}$$

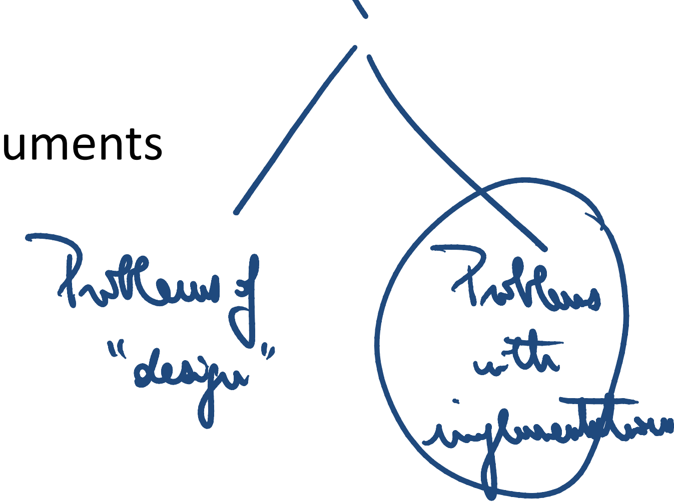
$$\pi_{01}=\frac{u_{01}}{u_{00}+u_{01}},\quad \pi_{11}=\frac{u_{11}}{u_{10}+u_{11}}$$

obs 1 obs 2
0 0
1 1
0 1
1 0

- Stress-testing
  - » Considers how portfolio would perform under extreme market moves
  - » Scenarios can be taken from historical data (e.g. assume all market variable move by the same percentage as they did on some day in the past)
  - » Alternatively they can be generated by senior management

Overview

- Model risk: models may be inappropriate because:
  - They do not reflect the true statistical behavior of the data
    - ✓ For normal market conditions
    - ✓ For extreme events
  - They can't be used consistently for special instruments
- Liquidity risk
- And after all, is VaR what you need?



Specific issues

- Fat tails
  - Student « t » distributions
  - Jump processes → Poisson process
- Time variation in risk: based on econometric studies
  - ARCH and GARCH models
  - Exponentially Weigthed Moving Average (EWMA) forecast
  - Regime switching
  - Dynamic correlations